Dual Heuristics with Graph Seach Depth-Limit Pruning: An Approach to the Personnel Tour Scheduling Problem

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1 Abstract

The aim of this research is to make an adaptable solution to the personnel scheduling problem under the robust tour scheduling problem formulation. Most solutions in the problem domain rely on mathematical reductions often within translations to linear representations that are not expandable to other domains. This paper aims to create a scheduling solution that can be expanded to a wide variety of scheduling problems within multiple industries and other resource allocation problems. As the number of shifts and employees increase in an organization, the number of possible shift-employee assignment configurations that form a completed schedule increases by a near factorial rate. The constraints of the organization also increases abstractly as the number of stakeholders (employees) increase as well. This paper hopes to describe a new perspective on the problem that allows for expansion and customization on the fly to fit changing, vague or arbitrary constraints. At the expense of computational efficiency, the author creates mechanisms that can be expanded or contracted to other industries and potentially other resource allocation problems outside of personnel scheduling. In highly variable or seasonally affected industries scheduling requirements can change rapidly between scheduling horizons. In the service industry, holidays and seasonal events can impact scheduling requirements that are difficult to incorporate to existing scheduling solutions quickly. This paper explores priority (rank) customization mechanisms that scheduling managers dealing with tour (shift/time period) scheduling problems could use to impact how the schedule is formed algorithmically. Within arbitrary heuristic priority rankings among both employees and shifts. This paper lays out a base algorithm for the single-role, single-location tour scheduling problems A dual heuristic approach is taken to select each choice, in the vast search space of potential configurations of shift-employee assignments. The dual heuristic approach addresses issues of other scheduling algorithms

which commonly simplify branching choices made in their solutions to one of either shifts or employees. At the loss of complete search space exploration.

2 Problem Description

Assigning many shifts to many full-time and part-time employees is a common problem faced by businesses, especially in the service industry and the problem is NP-Hard. Complexity is beyond polynomial and beyond feasible complete exploration. Managers often spend significant time trying to find an optimal configuration of full-time and/or part-time staff for a variable number of shifts on a week to week basis. Balancing complicated staffing needs and employee needs and preferences can be difficult for managers, where early assignment decisions can have a significant impact on the quality of the final schedule. Because of the significant search space in a selection of unique shifts and unique part-time employees a near optimal solution is difficult for inexperienced scheduling managers to find: a configuration that can fairly address employee availability and preferences as well as the requirements of the business. Different businesses have many different varying needs and requirements, so an algorithm will need to be flexible enough to handle a wide variety of constraints and preferences. The problem could abstractly be conceptualized as a task-resource allocation problem. Restaurants and other service occupations can have employees that are able to fulfill multiple roles in the planning horizon and may have particular requirements in the number of hours they prefer to be scheduled for an individual role. This helps restaurants maintain flexibility by cross-training employees that may be needed to fulfill other employee's duties and can be highly valuable in creating a cohesive workplace with fewer boundaries or divisions between roles and departments. This poses a challenge to many algorithms that do not account for this, this adds a layer of complexity that makes common scheduling algorithms difficult to adapt. Often in individual organizations the requirements have unique components that influence the decision making in assigning shifts to employees, such as the skill level of an employee, or the manager's priority of assigning certain shifts under limited staffing conditions. Many times these types of constraints could be represented by a relative value. In research no writings were found that include a concept of custom 'dialing' by the scheduler. Likely in an effort to limit mathematical complexity, and the limitations in evaluating results becoming closer to the subjective. Custom dials however are an important factor to scheduling managers, that employees who are more capable of fulfilling a shift should be ensured to be assigned before others, given limited shifts. Those employees should be more guarenteed to reach their indivudal requirements by the priority as well, in this simplified example they have a correlated importance to the scheduling manager.

In an example of 100 shifts that need to be assigned, 2 shifts per employee and with 50 employees to assign, the number of different configurations is 100!/2. Evaluating every configuration is not feasible in nearly any real-world scenario that addresses a problem of this size. Many works in personnel scheduling can avoid this complexity by avoiding one of the two choice types as a branching point, usually addressing the addition of a new pair of an employee and a shift as one choice. Most papers avoid addressing each as an individual choice. This can mathematically ensure a complete search and thus the optimal solution to the formulation, but this comes to the cost of detail by simplifying the problem.

3 Literature Review

3.1 Introduction

The objective of this literature review is to explore the existing research in the field of personnel scheduling, specifically focusing on solutions that can be applied to service jobs. Managers in highly variable industries, such as restaurants, often face changing staffing requirements, with shifts varying in duration and start times on a weekly basis. In these settings, both the shifts and employees can be unique, posing challenges in terms of problem complexity and flexibility. This review aims to identify similarities between existing studies, novel additions, and potential areas for further research in this field. Articles relevant to the research at hand are summarized and discussed in the following sections as they relate to generalization and adaptability of the problem formulation to service jobs particularly. The shift scheduling problem involves assigning staff to shifts over a planning horizon while ensuring that work rules and constraints are observed. Existing literature approaches this problem from different angles, with varying definitions of hard and soft constraints, problem formulations, and solution methods.

3.2 Research Article: Personnel Scheduling: A Literature Review

This literature review distinguishes three main groups of personnel scheduling: shift scheduling, days off scheduling, and tour scheduling, which combines the first two types. The complexity and size of these problems depend on factors such as the duration of the minimum planning interval. In this review, various solution methods are identified, including linear programming and construction-based approaches. The authors note that most personnel scheduling problems involve multi-objective models, with different stakeholders having their own priorities. This adds complexity to the problem, as the importance of each priority must be determined and combined into a single objective function. The authors point out that many approaches tend to simplify the problem at hand and make optimizations that prevent the solution for being generalized to other settings. This highlights the need for adaptable and flexible solutions that can be applied to a wide variety of industries.

3.3 Research Article: A Graph-Based Formulation for the Shift Rostering Problem

This paper presents a graph-based formulation for the shift rostering problem, focusing on a generalized approach that can be adapted to different application areas and accommodate changes within an organization. The authors propose a method that models coverage requirements as soft constraints and work rules as hard constraints. They argue that the shift rostering problem is challenging due to the large-scale mixed-integer programs and work rule constraints, which become even more complex when considering staff heterogeneity. Their graph-based formulation takes into account hard and soft requirements, allowing for a flexible and an adaptable solution. Their solution does not account for heterogeneity among employees and so fails to be adaptable to a wide variety of industries that require this.

3.4 Research Article: An Integer Linear Programming-Based Heuristic for Scheduling Heterogeneous, Part-Time Service Employees [3]

This research article proposes a two-stage procedure for solving the scheduling problem using a linear program to create potential shifts and a heuristic based linear programming approach to assign these shifts to employees. The authors elaborate that their algorithm finds the employee with the fewest number of shifts that could be assigned and then chooses the set of shifts for the planning period all at once for that employee. This approach aims to provide a flexible and efficient solution to the scheduling problem, with potential for further optimization and generalizability. The authors use clever data orientation that helps computational efficiency by forming the problem into a linear program with mathematical reductions. These efficiencies do not open themselves to expansion easily. This paper and many others in scheduling algorithm research give optimizations that help the specific problem domain and constraints but do not fit themselves to adaptation and expansion. While being the most applicable to restaurant service staff of modern papers, it does not do enough to expand a generalization.

3.5 Literature Discussion and Conclusions

Personnel scheduling problems often involve widely different problem formulations, as each organization has unique constraints and requirements. This makes comparison between methods challenging and highlights the need for adaptable and flexible solutions. Restaurants, for example, require an approach that allows for overlapping shifts, flexible start times, and variable shift lengths, among other constraints. The variety of approaches shows that a comprehensive comparison between the vari- ous methods presented in the literature could enhance the understanding of the personnel scheduling problem and facilitate the development of more adaptable solutions. For in- stance, examining the trade-offs between graph-based formulations and linear programming approaches in terms of computational efficiency, ease of implementation, and flexibility in accommodating diverse constraints would be valuable. Furthermore, exploring the integration of machine learning and artificial intelligence techniques, along with the heuristic approaches, might present novel solutions for handling complex and heterogeneous scheduling scenarios. The tour scheduling approach combines traditional shift scheduling and days off scheduling. These problems are typically solved using a combination of heuristics, machine learning, and operations research. They have numerous practical applications, such as logistics, supply chain management, and transportation outside of personnel scheduling explicitly.

Some research exists in this topic but is limited to specific problem formulations. The use of AI and ML techniques could lead to more robust and flexible systems in a generalized approach but introduces more complexity. Solutions exist that are capable of addressing the unique requirements of service jobs, such as restaurants, while maintaining an optimal balance between staff preferences and organizational needs, but do not address many types of operational uncertainty. Overall, a deeper investigation into the existing literature and a broader inclusion of relevant studies would help to identify the strengths and weaknesses of different methods, paving the way for the development of more effective and adaptable personnel scheduling solutions. Existing literature offers a range of methods for addressing the shift scheduling problem, from graph-based formulations to linear programming and heuristic approaches. While these methods provide valuable insights into potential solutions, there remains a need for more research into the development of accessible, drag-and-drop solutions for personnel scheduling, taking into account constraints and their weighted importance according to the schedule decision maker and the applicable stakeholders.

Existing literature on personnel scheduling offers a variety of approaches to address the challenges faced by managers in industries with highly variable staffing requirements. However, there remains a need for more research into the development of adapt- able, flexible, and accessible solutions that can accommodate the unique constraints. Each scheduling problem tends to have properties that are unique to the organization, making it difficult to generalize solutions. Restaurants and other service jobs require a solution that can be adapted to a wide variety of constraints, and exploring the use of heuristic approaches could provide a flexible and efficient solution to other scheduling problems. Research that factors in customization that add an amount of ongoing tweaking by the primary scheduling manager and other stakeholders is rarely explored. Survey research and elicitation of managers or the employees they schedule is difficult to find and so the goals of research may not align with what is needed or preferred on the ground.

4 Approach - Data Representation and Algorithm

Consecutive days off in this problem formulation can be solved by predetermining each employee's days off and including the time period as unavailable for the employee to be scheduled. This doesn't allow for as much flexibility in problem instances where the employee's days off are less strictly adhered to consistently between planning horizons. This solution bluntly answers the question of intra-horizon consecutive days-off. For example, instances where the last day of a planning horizon and the first day of the next planning horizon could both be selected as days-off for a valid configuration of an employee's schedule. This solution addresses consecutive days-off in a given horizon in the same mechanism that says if an employee is available for a shift. The employee's availability data for some 30min schedule slot.

In choosing employee-shift assignments. The order in which shifts and employees are chosen will play a significant role in the quality of the solution. With a particular case in mind of scheduling managers wanting to find an optimal schedule for a week-long period planning horizon.

Unique to the formulation of this problem, shifts are unique and have an arbitrarily set priority level that can be compared amongst them, so the shift with the highest priority is going to be a more advantageous choice over others in end score, all else held equal. Likewise employees are given a priority in kind. This could be thought of the scheduling manager's idea of how important scheduling this shift before others is and for employees as a ranking based on time with the organization or their skill level. For the formulation of the problem: shifts and employees will be each given an abstract priority level, this will be used to essentially scale other factors when scoring and comparing shifts or employees to be selected. The core concept is that these are arbitrary ranking values that the manager could adjust or append that will result in schedule configurations that fit the change that the manager made. An example representation of this could be a skill level change of an employee the manager inputs. So in the assignment of shifts, that employee is of a higher priority to fit the employee's scheduling requirements and simultaneously a more advantageous employee to schedule because of their skill level. This problem formulation assumes that these factors move in tandem. The scheduling manager or algorithm is incentivized to schedule the employee more, and thus they are more likely to have their minimum requirements met as well.

In this formulation Employees are unique where their requirements, and availability can differ widely. The availability of employees could be as granular as desired, where the employees may be available for morning, afternoon or night shifts for example or an approach where 30min periods are categorized as available or unavailable for that employee. Effectively when trying to find the potential employees available for a shift, planning for a granular 30 min slot approach will solve a less granular availability problem just the same if translated to the data structures of this algorithm. Employees are assigned their arbitrary priority level by a manager and this is standardized from 0-1 among employees. Again, this could be used as a representation of the skill of an employee in their role and could contribute to the manager's inclination to select that employee for a certain shift in a manual approach to forming a schedule. When set as a custom value by the manager, these rankings can be included in heuristics such that the resulting schedule configurations follow that priority all else held equal.

A scoring mechanism that assesses a partial or complete solution, a configuration of shifts and employees, will be used to attempt a standard optimization metric. That scoring metric will be used to compare and select from configurations when pruning solution branches. If the scoring metric is constructed with a requirement of the ability of scoring a partially completed schedule, the metric can be used as well to score a completed schedule. The scoring metric in whole will be made from two components put into ratios. So the score can then be compared in a standard form with values between 0-2 non-inclusive. Where a lower score represents a schedule configuration that better fits the constraints. A score of 2 would then be a schedule configuration where no shifts are assigned and every employee has 0 hours scheduled. High priority employees are less likely to be under their hour minimum and high priority shifts are less likely to be unassigned in solutions with a lower score. Schedules formed under the same custom set of arbitrary priorities for shifts and employees are comparable.

The approach of the algorithm will be to use a dual heuristic scoring method with depth based pruning by score assement of partial configuration soltions constructed. 2 heuristics will be constructed, one for choosing the next best shifts to assign and another for choosing the most fit employees to assign to a selected shift. After an employee is assigned to the shift, the state of the schedule is updated and passed to the next shift selection branches. The top b of unassigned shifts, and top b employees to assign to each of those shifts will be selected and make b branches at each selection. The next best n shifts to assign will be chosen, given the current configuration, and then the next best n employees to assign to those shifts will be chosen. When we make the choice of which shift to select, or which employee to assign, we can consider that choice as a branch in the search tree. Both decisions make impacts on the final configuration, so both decisions should be considered as branches from those choices where shifts and employees are both unique.

5 Experiments and Results

The algorithm described above will use the solution score mechanism described below. The solution score and the heuristics for shift selection and employee selection are all designed to be able to be expanded on with standardized ratios influenced by the custom priority rankings a manager may set, within the algorithm described above the scoring and heuristic methods could be altered by adding extra metrics that could be unique to an organization. The algorithm above was designed with expansion in mind, but only the base case is explored in this paper, so the scoring metrics for the solution and heuristics will be unique to the scheduling problem at hand.

For this research a custom Python script was developed that conducts the data engineering and forms partial solutions expanded and then pruned by the algorithm described in this work.

For all shifts a priority is set at random, that priority level will remain constant among trials and is used as a constant factor to be used in comparing results of the independent trials conducted.

Each of the scoring metrics among the solutions score, shift selection heuristic score and employee selection heuristic score are all designed as 'lower is better' for a uniform optimization method.

5.1 Schedule Configuration - Solution Score

The schedule configuration solution ratio components are below. The two ratios are added together to form the solution score, which is already standardized in parts to a value between 0-2 in summation.

1. shift hours unassigned - ratio of:

(a) total (shift priority weighted) hours of unassigned shifts

(b) total (shift priority weighted) hours of all shifts.

2. employee hour minimums met - ratio of:

    (a) total (employee priority weighted) hours under employee minimums max(0, employee min hours - employee scheduled hours)

    (b) total (employee priority weighted) employee min hours.

$$Score = \frac{S_u}{S} + \frac{E_u}{E} \tag{1}$$

5.2 Shift Selection - Shift Heuristic Score

Multiplication of the following components:

1. Available Employee Ratio - Ratio of:

    (a) Employees unavailable for the shift, by employee availability and their scheduled shifts in the current solution.

    (b) Total number of employees in the pool that can fulfill this shift assignment.

2. Shift Priority

$$H_S = \frac{E_U}{E} * S.priority \tag{2}$$

5.3 Employee Selection - Employee Heuristic Score

Summation of the following components:

1. Ratio of: hours scheduled under hours minimum over minimum hours. Weighted by employee priority.

2. Ratio of: Hours scheduled in excess over maximum hours as a ratio of total hours weighted by employee priority

$$H_E = E_p * (\frac{h_s}{h_m} + \frac{h_e}{h_m}) \tag{3}$$

5.4 Experiment Design

As a jumping off point, an anonymized past and completed schedule of an active restaurant was evaluated for use as control data. [CENTRO] This completed schedule is used to form control data. The control data was limited to a single week of scheduling, within a single role and location.

36 shifts and 11 employees are available in the control data.

In the sample data consecutive days off are built into each of the employee's availability. We will assume that this is a requirement of the scheduling problem and that the employee's days off are not flexible within the planning horizon.

A basic extrapolated availability is made for each employee from their scheduled shifts in the sample schedule data. The availability schedules of individual employees must be extrapolated from the hours they were assigned in the sample schedule data. This will act as the control data for the experiment. The control data is useful for providing a theoretical best case scenario for the algorithm to attempt to match, but does not account for the variability in employee availability in practice. Employee availability is often more flexible than the control data would suggest. Past the control, we will use an expanded control data set.

The control data is expanded to attempt to make a more realistic availability dataset that provides more variability in the search space, without being too far from the control data. We would expect the expanded control data to show a difference in the results of the algorithm, but not too far from the control data. Within the expanded control data, the daily availability for each employee is added an hour on either end of their start and end times.

Held constant among trials are the unique shifts. The priority levels of shifts and employees are also held constant through each of the trials. If an employee's data is duplicated for a trial, the priority level of the employee is also duplicated and a choice of the duplicate would be indistinguishable from the original employee.

The arbitrary shift and employee priorities are set at random uniquely and held constant among trials for the number of shifts and number of employees respectively. The priority levels are set at random among equal intervals from 0-1 non-inclusive. If there are 20 employees or shifts, the highest priority of each is intended to be the least valuable to assign, all else held equal.

The control trial (1) is conducted on the exact availability determined by the shifts that each employee was assigned in the planning horizon.

An 'expanded' control is made to attempt a more realistic example of employee availability.

The 'expanded control' extends the availability of employees in the base control case by 1 hour on either end of their scheduled shifts.

The pool of shifts that should be assigned is held constant through each of the 4 following trials.

1. Control case - Employee Availability is exactly determined by the shifts the employee was scheduled in the sample data.

2. Expanded Control case - Employee Availability is expanded 1 hour (two 30 min slots) from both the start and end of the shifts that the employee was assigned in the sample.

3. Understaffed cases - Adapted from the Expanded Control, mean results among 10 random samples of employee data contractions. Delete 2 random employees from the data while ensuring each trial is unique.

4. Overstaffed cases - Adapted from the Expanded Control, mean results among 10 random samples of employee data expansions. Add 2 different random copies of employees in the data.

5.5 Experiment Results

Trial	Solution Score (0-2)	Run Time (min)	Coverage (hrs / total hrs)
1	0.0502	2.52	100%
2	0.0000	2.56	100%
3	0.2248	2.30	92%
4	0.1304	2.62	100%

6 Evaluation

Under tight constraints to be met, the algorithm is able to find a solution that is comparable to the control data. Within the control trial, the algorithm was unable to match the exact solution of the control data, but was able to find a solution that was comparable in score, notably one employee was scheduled above their maximum requirements. Potentially poor choices are made early that prevent the algorithm from finding a solution that is as good as the control data provides in a manual capacity. The expanded control data shows a result that suggests the algorithm is able to find a solution that is comparable to the control data, but with more variability in the search space.

The scores of over and understaffed trials are higher than the control data. Suggesting that the understaffed trial fails to assign shifts to high priority employees. The overstaffed trial assigns the lowest priority employees to shifts reaching over their maximum requirements. In the overstaffed trial, those employees that are scheduled over their maximum would contribute less to the score than the understaffed trial. The overstaffed trial is able to assign shifts to high priority employees, but the score is still higher than the control data.

The time of each run is comparable, but the algorithm is not optimized for speed. This algorithm would need work before being applied to larger problems.

7 Conclusion

The sample data is limited, and methods of generating feasible employee availability data are not explored in this paper. Generating possible datasets that can help to evaluate this algorithm and other personnel would be helpful to provide more cases to find potential pitfalls.

The score components: Each of these standardized factors could be given a custom set weight by a manager. This would maintain that schedules are comparable between the same initial conditions and increase control of a scheduling manager. In the factors we use, a manager could easily alter prioritization of shift hours assigned, or employee requirements being met. If a factor is more important for the business, the manager could tweak the weights of these scoring factors. But for this paper, the weights are equal. This structure however of ratios does allow for more ratio score factors to be added in problems expanded from what we describe here.

The problem type chosen here is without multiple roles individual employees could fulfill, but the problem structure in practice may require multiple roles within the same planning horizon and scheduling problem within an organization. The roles are scheduled together from the same pool of employees with one to many roles. The shifts are then also of one to many role types in kind.

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